Intensionality is Comonadic

New Type-Theoretic Tools in Natural Language Semantics

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Outline

Comonads

- Dual of monads
- Whereas monads are a tool for encoding semantic representations with ‘enriched outputs’, comonads do the same for ‘enriched inputs’. Enriched inputs could include:
    - Intensions
    - Contextually-provided information

We will focus on intensionality, particularly in the tradition of Montague.

Montague's *Intensional Logic*

- System combining higher-order modal logic with an 'intension' operator
- Due to Richard Montague in the 1960's (Montague 1973, Gallin 1975)
- Motivated by denotational semantics of natural language
- Gives logical forms for modal **de re** and **de dicto** sentences
- Interprets "intensional" natural language words and phrases: "former", "alleged", "necessarily"/"must", "is rising" ...

Montague's *Intensional Logic* (Modern Perspective)

- The intension operator has the structure of a comonad
- Earliest comonadic modal type theory (Awodey, Buchholtz, and Zwanziger 2015)
- Lessons from syntax of Bierman and de Paiva (1999), Pfenning and Davies (2000)
- Interpretation in a comonad on a topos (Awodey, Buchholtz, and Zwanziger 2016)

Outline

Problem: Modal De Dicto and De Re

Montague 1973

"The president is necessarily a politician."

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De Re: $(\lambda x.\Box \mathrm{Pol}(x))p$

Problem: Since we want to represent both *de re* and *de dicto*, we cannot have $(\lambda x.\Box \mathrm{Pol}(x))p \equiv \Box \mathrm{Pol}(x)[p/x] \equiv \Box \mathrm{Pol}(p)$!

Proposal: Modal De Dicto and De Re

Zwanziger (2017)

"The president is necessarily a politician."

De Dicto: $\Box \mathrm{Pol}(p)$

De Re: $\Box \mathrm{Pol}[p]$

Note that $(\lambda x.\Box \mathrm{Pol}[x])p \equiv \Box \mathrm{Pol}[x][p/x] \equiv \Box \mathrm{Pol}[p]$

Problem: Quantifying in

Quine (1943)

Failure of Existential Generalization

"The president is necessarily a politician." (De dicto) $\not\models$
"Someone is necessarily a politician" (De Re)

$\Box \mathrm{Pol}(p)$ (De dicto) $\not\models \exists x. \Box \mathrm{Pol}(x)$ (De Re)

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Quine famously concludes that $\exists x.\Box\mathrm{Pol}(x)$ does not make sense.

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Quine (1953):

"[If] to a referentially opaque context of a variable we apply a quantifier, with the intention that it govern that variable from outside the referentially opaque context, then what we commonly end up with is unintended sense or nonsense...In a word we cannot in general properly *quantify into* referentially opaque contexts."

Partial Solution: Quantifying in

Montague (1973)

- Allows quantifying into modal contexts, e.g. $\exists x.\Box \mathsf{Pol}(x)$
- However, abandons existential generalization rule when this would result in quantifying in

Proposal: Quantifying in

Zwanziger (2017)

- Existential generalization just works
- The principle $\Box\text{Pol}[p] \vdash \exists x.\Box\text{Pol}[x]$ is valid.
- The term $\exists x.\Box\text{Pol}(x)$ is ill-typed, so there is no question of $\Box\text{Pol}(p) \vdash \exists x.\Box\text{Pol}(x)$!
- The present proposal has both quantifying in and the usual rules for $\exists, \forall, \lambda$!

Intensionality

Montague (1973)/Zwanziger (2017)

- $[\![\text{elderly}]\!] : (\mathsf{E} \to \mathsf{T}) \to \mathsf{E} \to \mathsf{T}$
- $[\![\text{former}]\!] : \flat(\mathsf{E} \to \mathsf{T}) \to \mathsf{E} \to \mathsf{T}$

Intensionality

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- $[\![\text{former}]\!] : \flat(\mathsf{E} \to \mathsf{T}) \to \mathsf{E} \to \mathsf{T}$
- $[\![\text{is a politician}]\!] : \mathsf{E} \to \mathsf{T}$
- $[\![\text{is rising}]\!] : \flat\mathsf{E} \to \mathsf{T}.$

Need the intension operator $\hat{\ }$ to give access to global information!

	Syntax for Intension Operator (Adapted)	
	De Dicto	**De Re**
Montague (1973)	$\hat{}\, t(u)$	$(\lambda x.\hat{}\, t(x))(u)$
Bierman and de Paiva (1999)	$\hat{}\, t(u)$ with $\cdot$ for $\cdot$	$\hat{}\, t(x)$ with u for x
Pfenning and Davies (2000)	$\hat{}\, t(u)$	let $\hat{}\, x = u$ in $\hat{}\, t(x)$
Zwanziger (2017)	$\hat{}\, t(u)$	$\hat{}\, t([u])$

Figure: Comparison of Comonadic Modal Type Theories

Types

- There are basic types E, T.
- If A and B are types, then so is $A \to B$.
- If A is a type, then so is $\flat A$.

Term Calculus

Logic

- $\dfrac{}{\cdot \vdash \top : \mathsf{T}}$, and similarly for $\bot$
- $\dfrac{\phi : \mathsf{T} \qquad \psi : \mathsf{T}}{\phi \wedge \psi : \mathsf{T}}$, and similarly for $\vee$ and $\Rightarrow$.
- $\dfrac{\phi : \mathsf{T}}{\neg\phi : \mathsf{T}}$
- $\dfrac{\Gamma, x : A \vdash \phi : \mathsf{T}}{\Gamma \vdash \forall x.\phi : \mathsf{T}}$, and similarly for $\exists$.
- $\dfrac{t : A \qquad u : A}{t =_A u : \mathsf{T}}$

Term Calculus

Variables and Function Types

- $$\frac{}{\Gamma, x : A, \Delta \vdash x : A}$$
- $$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \to B}$$
- $$\frac{t : A \to B \qquad u : A}{tu : B}$$

Note that we will have: $\lambda x.(tx) \equiv t \qquad (\lambda x.t)x' \equiv t[x'/x]$

Term Calculus

Modality

($\flat$ Intro., or Intension).

$$\frac{\Gamma \vdash s_1 : \flat A_1, \ldots, \Gamma \vdash s_n : \flat A_n \qquad x_1 : \flat A_1, \ldots, x_n : \flat A_n \vdash t(x_1, \ldots, x_n) : B}{\Gamma \vdash \; ^\frown t([s_1], \ldots, [s_n]) : \flat B}$$

($\flat$ Elim., or Extension).

$$\frac{\Gamma \vdash t : \flat A}{\Gamma \vdash \; ^\smile t : A}$$

(Box Form.).

$$\frac{\Gamma \vdash s_1 : \flat A_1, \ldots, \Gamma \vdash s_n : \flat A_n \qquad x_1 : \flat A_1, \ldots, x_n : \flat A_n \vdash \phi(x_1, \ldots, x_n) : \mathrm{T}}{\Gamma \vdash \Box\phi([s_1], \ldots, [s_n]) :\equiv \; ^\frown \phi([s_1], \ldots, [s_n]) =_{\flat \mathrm{T}} \; ^\frown \top : \mathrm{T}}$$

Outline

Definition of a Comonad

Computer Science Version

A **comonad** on a topos $\mathcal{E}$ consists of

- a function $\flat : \mathcal{E} \to \mathcal{E}$
- an arrow $\varepsilon_A : \flat A \to A$ for each $A \in \mathcal{E}$
- an arrow $f^* : \flat A \to \flat B$ for each arrow $f : \flat A \to B$

such that:

- $\varepsilon_A^* = \mathrm{id}_{\flat A}$ for each $A \in \mathcal{E}$.
- $\varepsilon_B f^* = f$ for each $f : \flat A \to B$.
- $g^* f^* = (g(f^*))^*$ for each $f : \flat A \to B$ and $g : \flat B \to C$.

Let T be a theory of MIL. An **interpretation** of T consists of:

- A topos $\mathcal{E}$
- A finite limit-preserving comonad $\flat : \mathcal{E} \to \mathcal{E}$
- An object E of $\mathcal{E}$
- An interpretation function $[\![-]\!]$ defined on types and terms derivable from T...

Interpretation

- $[\![\flat A]\!] = \flat[\![A]\!]$

Interpretation

- $[\![\flat A]\!] = \flat[\![A]\!]$
- If $x_1 : \flat A_1, ..., x_n : \flat A_n \vdash t : B$ is a term, then $[\![\,\hat{}\,t([x_1], ..., [x_n])]\!]$ is

$$[\![\flat A_1]\!] \times ... \times [\![\flat A_n]\!] \xrightarrow{[\![t]\!]^*} \flat[\![A]\!] \quad .$$

Interpretation

- $[\![\flat A]\!] = \flat [\![A]\!]$
- If $x_1 : \flat A_1, \ldots, x_n : \flat A_n \vdash t : B$ is a term, then $[\![^\frown t([x_1], \ldots, [x_n])]\!]$ is

$$[\![\flat A_1]\!] \times \ldots \times [\![\flat A_n]\!] \xrightarrow{[\![t]\!]^*} \flat [\![A]\!] \quad .$$

- If $\Gamma \vdash t : \flat A$ is a term, then $[\![^\smile t]\!]$ is

$$[\![\Gamma]\!] \xrightarrow{[\![t]\!]} \flat [\![A]\!] \xrightarrow{\varepsilon_{[\![A]\!]}} [\![A]\!] \quad .$$

Outline

Interpretation

- $[\![\flat A]\!](w) = \prod_{w' \in W} [\![A]\!](w')$
- $([\![\, \smallsmile \,]\!](w))(i) = i(w)$
- $[\![\, \frown c]\!](w) = \lambda w'.[\![c]\!](w')$
- $([\![\, \frown t]\!](w))(a) = \lambda w'.([\![t]\!](w'))(a)$

Other Examples

- Kripke semantics with an arbitrary S4 accessibility relation
- Boolean-valued models (as in Gallin 1975)
- …

Outline

Combining DTT and Comonads

- Comonadic Modal Dependent Type Theory (Zwanziger 2018)
  - Hyperintensional
  - Can integrate other work on DTT

Intensional Martin Löf Dependent Type Theory

$+$

Montague Intensional Logic

$=$

Hyperintensional Type Theory (!!)

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