Formal Semantics in Modern Type Theories

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Structure

- What are MTTs?
- Brief intro to MTTs
- Examples of using MTTs for NL semantics
- Conclusions

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Modern Type Theories

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    - ⋆ Rich typing

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    - ★ Type universes
    - ★ Proof theoretic specification

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  - Alternative foundations for mathematics (Homotopy Type Theory) [Voevodsky(2015)]
  - Formalization using proof assistants: systems implementing constructive type theories that help in the formalization of mathematics and program verification
    - ★ prime examples: Agda [Agda 2008()], Coq [Coq 2007()]

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Modern Type Theories and linguistic semantics

- Starts with the seminal work by Ranta [Ranta(1994)] and earlier (e.g. Sundholm [Sundholm(1989)])

Modern Type Theories and linguistic semantics

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  - Many more after that [Boldini(2000), Cooper(2005), Dapoigny and Barlatier(2009), Bekki(2014), Retoré(2013), Grudzinska and Zawadowski(2014), Chatzikyriakidis and Luo(2012), Chatzikyriakidis and Luo(2017a)] among others
  - How they are useful and in what ways they are different from STT?

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Basic Types: Rich Typing

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- In MTTs, no such restriction exists: the universe of entities CAN be many-sorted
  - Arbitrary number of types can be available giving more structure to the domain of individuals, e.g. $man, chair : Type$ (this is the approach by Ranta, Boldini, Luo and colleagues among others)
    - ★ This is known as the CNs-as-Types approach [Chatzikyriakidis and Luo(2016 (to appear).)]
    - ★ However, this is a choice! Other researchers like Bekki and colleagues working on MTTs, prefer to interpret CNs more standardly, i.e. as predicates [Bekki(2014)]

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    MS man: $e \rightarrow t$

    MTTs man: Type

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Basic Types: Rich Typing

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Basic Types: Rich Typing

- Selectional restrictions as type mismatch: the ham sandwich talks
  - Talk: $human \rightarrow Prop$
  - the ham: ham (with $ham : Type$)
  - Functional application not possible!

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Subtyping

- A further consequence of a rich selection of types
  - Subtyping mechanism: otherwise the system becomes too rigid
  - Even things like *the man walks* would not be possible with no subtyping mechanism
    - $walk : Animal \rightarrow Prop$
    - $the\_man : Man$ (with $man : Type$)
    - Fine if $man \leq human$

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Different Systems of Subtyping

- Classic case: Subsumptive subtyping

$$\frac{a{:}A, A \leq B}{a{:}B}$$

- a term of type A can be used in a context where a term of type B is required instead just in case $A \leq B$

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Different Systems of Subtyping

- Record Type Subsumption: a type of subsumptive subtyping for TTR

$$\begin{bmatrix} x & : & Man \\ y & : & Donkey \\ e & : & own(x,y) \end{bmatrix}$$

will also be of type

$$\begin{bmatrix} x & : & Man \\ y & : & Donkey \end{bmatrix}$$

and also of type

$$\begin{bmatrix} x & : & Man \end{bmatrix}$$

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    - ★ A is a (proper) subtype of B ($A < B$) if there is a unique implicit coercion c from type A to type B
    - ★ An object a of type A can be used in any context $\mathfrak{C}_B[\_]$ that expects an object of type B: $\mathfrak{C}_B[a]$ is legal (well-typed) and equal to $\mathfrak{C}_B[c(a)]$.

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- Metatheoretically more advantageous: canonicity is preserved
  - Long story!

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Complex Types and Dependent Typing

- STT involves basic types and function types constructed out of the basic types

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Complex Types and Dependent Typing

- STT involves basic types and function types constructed out of the basic types
- MTTs offer a range of other more advanced typing structures
  - Dependent Typing
    - ★ A family of types that may depend on some value

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Complex Types and Dependent Typing

- Dependent Types Π and Σ
  - When A is a type and P is a predicate over A, $\Pi x{:}A.P(x)$ is the dependent function type that stands for the universally quantified proposition $\forall x{:}A.P(x)$
  - Π for polymorphic typing: $\Pi A{:}CN.(A \rightarrow Prop) \rightarrow (A \rightarrow Prop)$

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  - Π for polymorphic typing: $\Pi A{:}CN.(A \rightarrow Prop) \rightarrow (A \rightarrow Prop)$
  - A is a type and B is an A-indexed family of types, then $\Sigma x{:}A.B(x)$, is a type, consisting of pairs (a, b) such that a is of type A and b is of type $B(a)$.
  - Adjectival modification as involving Σ types [Ranta(1994), Chatzikyriakidis and Luo(2017b)]: $[\![heavy\_book]\!] = \Sigma x : book.heavy(x)$

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Intro to MTTs-Universes

- Universes
  - A universe is a collection of (the names of) types into a type (Martin Löf, 1984).

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  - A universe is a collection of (the names of) types into a type (Martin Löf, 1984).
  - Universes can help semantic representations. For example, one may use the universe $\text{CN} : \mathit{Type}$ of all common noun interpretations and, for each type A that interprets a common noun, there is a name $\overline{A}$ in CN. For example,

$$\overline{man} : \text{CN} \quad \text{and} \quad T_{\text{CN}}(\overline{man}) = man.$$

    In practice, we do not distinguish a type in CN and its name by omitting the overlines and the operator T_{CN} by simply writing, for instance, $man : CN$.

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Intro to MTTs-Universes

- Universe of linguistic types (LType) [Chatzikyriakidis and Luo(2012)]

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- Universe of linguistic types (LType) [Chatzikyriakidis and Luo(2012)]
  - Introduced to deal with conjoinable types
  - A universe over which the coordination operator extends

$$\frac{}{PType : Type} \qquad \frac{}{Prop : PType} \qquad \frac{A : LType \quad P(x) : PType\ [x{:}A]}{\Pi x{:}A.P(x) : PType}$$

$$\frac{}{LType : Type} \qquad \frac{}{\mathrm{CN} : LType} \qquad \frac{A : \mathrm{CN}}{A : LType} \qquad \frac{A : PType}{A : LType}$$

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Contexts

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Contexts

- Context in type theory is a formal notion
  - Various way of thinking about contexts
    - List of variable declarations, where variables stand for proofs of the corresponding assumptions
    - A sequence of type judgements
    - Formally, a context is an expression of the form:

$$\Gamma = x_1 : A_1, x_2 : A_2(x_1), \ldots, x_n : A_n(x_1, \ldots, x_{n-1})$$

    - A series of types, and a series of proof objects for these types
    - Any type may depend on any of the previous proof objects

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Contexts

- They have been used instead of possible worlds for belief intensionality [Ranta(1994), Chatzikyriakidis and Luo(2013)] and also to formalize discourse structure [Ranta(1994), Boldini(2000), Chatzikyriakidis and Luo(2014)]

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- Consider the following discourse:

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- Following the end of the first sentence, we have:

  $$x_1 : (\Sigma x : Farmer)(\Sigma y : Donkey)(own(x, y))$$

- The pronouns pick variables already declared using the projection operators (π_1 and π_2)

  $$x_2 : (love(\pi_1(x_1), \pi_1(\pi_2(x_1)))$$

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MTTs in action: Adjectives

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  - For example, from $\Sigma(man, black)$ it does not follow that something is black or that something is a black human
- [Chatzikyriakidis and Luo(2017b)] and earlier work: Extend the account with subtyping and polymorphic types

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  - Regular type for intersective adjectives
    - $handsome : human \to Prop$
  - polymorphic for subsectives
    - $skilful : \Pi A : \mathrm{CN}.\ (A \to Prop)$

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  - $\Sigma(man, black) \leq Man$

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- The first projection π_1 of the Σ is declared as a coercion
  - $\Sigma(\mathit{man}, \mathit{black}) \leq \mathit{Man}$
    - ★ Thus, from black man we can infer man
  - Subtyping propagates through the constructors: if $A \leq B$ then forall $P{:}C \rightarrow \mathit{Prop}$ (with $A, B \leq C$), $\Sigma(A, P) \leq \Sigma(B, P)$
    - ★ This means that: $\Sigma(\mathit{man}, \mathit{black}) \leq \Sigma(\mathit{human}, \mathit{black})$

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  - Polymorphic type restricted to the CNs class
  - The modification involves an argument which is the class restriction, so $\Sigma(\mathit{surgeon}, \mathit{skilful}(\mathit{surgeon}))$
    - ★ It does not follow that a skilful surgeon is a skilful human: $\Sigma(\mathit{surgeon}, \mathit{skilful}(\mathit{surgeon})) \not\Rightarrow \Sigma(\mathit{human}, \mathit{skilful}(\mathit{surgeon}))$
    - ★ $\Sigma(\mathit{human}, \mathit{skilful}(\mathit{surgeon}))$ is not well-typed, $\mathit{skilful}(\mathit{surgeon}){:}\mathit{surgeon} \rightarrow \mathit{Prop}$ amd our π_1 is of type *human*

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- A belief context: a sequence of judgments a specific human holds
  - $\Gamma_p = x_1 : A_1, ..., x_n : A_n(x_1, \ldots, x_{n-1})$

MTTs in action: Adjectives

- Intensional adjectives: alleged
- A belief context: a sequence of judgments a specific human holds
  - $\Gamma_p = x_1 : A_1, ..., x_n : A_n(x_1, \ldots, x_{n-1})$
  - We can similarly define allegation contexts
  - Let $A_N : \mathrm{CN}$ is the interpretation of a common noun N. Then: $alleged\ N = \Sigma a : Human.\ \Gamma_a(A_N)$
  - An alleged N: has been alleged by someone that it is an N
    - Belief and allegation contexts can be kept separate

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Adjectival Modification/Multidimensional Adjectives

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  - The antonyms of these type of multidimensional adjectives existentially quantify over dimensions

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  - We define an inductive type *health*
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  - We define an inductive type *health*
    - $Inductive\ [\![Health]\!] : D :=Heart \mid Blood\_pressure \mid Cholesterol$
  - Then we define:
    - $healthy = \lambda x{:}Human.\forall h{:}Health.Healthy(h)(x)$
    - $sick = \lambda x{:}Human.\neg(\forall h{:}Health.Healthy(h)(x))$
- For gradability and gradable adjectives have a look at [Chatzikyriakidis and Luo(2017a)]

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Adverbial Modification

- Typing issues: How are we going to type adverbs in a many sorted TT?

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Adverbial Modification

- Typing issues: How are we going to type adverbs in a many sorted TT?
  - Two basic types
    - ★ Sentence adverbs: $Prop \rightarrow Prop$
    - ★ VP-adverbs: $\Pi A{:}CN.(A \rightarrow Prop) \rightarrow (A \rightarrow Prop)$
    - ★ Polymorphic type: Depends on the choice of A
    - ★ Given that we are talking about predicates, depends on the choice of the argument
    - ★ $walk{:}Animal \rightarrow Prop \Rightarrow ADV(walk){:}(Animal \rightarrow Prop)$
    - ★ $drive{:}Human \rightarrow Prop \Rightarrow ADV(drive){:}(Human \rightarrow Prop)$

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Adverbial Modification: Veridicality

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Adverbial Modification: Veridicality

- Veridical Adverbials when applied to their argument, imply their argument
  - John opened the door quickly $\Rightarrow$ John opened the door
  - Fortunately, John is an idiot $\Rightarrow$ John is an idiot
- Non-veridical adverbs do not have this property
  - John allegedly opened the door $\not\Rightarrow$ John opened the door

Adverbial Modification: Veridicality

- Define an auxiliary object first, then define the adverb as its first projection
  - $VER_{Prop} : \Pi v : Prop. \, \Sigma p : Prop. p \supset v$
  - $ADV_{ver-Prop} = \lambda v : Prop. \, \pi_1(VER_{Prop}(v))$
- An adverb like *fortunately* will be defined as:
- $fortunately = \lambda v : Prop. \, \pi_1(VER_{Prop}(v))$

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  - $ADV_{ver-Prop} = \lambda v : Prop.\ \pi_1(VER_{Prop}(v))$
- An adverb like *fortunately* will be defined as:
- $fortunately = \lambda v : Prop.\ \pi_1(VER_{Prop}(v))$
- Consider the following: Fortunately, John went $\Longrightarrow$ John went
  - The second component of $(VER_{Prop}(v))$ is a proof of $fortunately(v) \Rightarrow v$
  - Taking v to be *John went*, the inference follows

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Adverbial Modification: Intensional/domain adverbials

- Use of TT contexts in this case as well
  - $allegedly = \lambda P : Prop.\ \exists p{:}human, A_p(P)$
  - Someone has alleged that P (A_p is an agent's allegation context [Chatzikyriakidis and Luo(2017b)]

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Adverbial Modification: Intensional/domain adverbials

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  - Someone has alleged that P (A_p is an agent's allegation context [Chatzikyriakidis and Luo(2017b)]
- Introduction of intenTional contexts: Contexts including the intentions (rather than the beliefs) of an agent. We can use this idea for adverbs like intentionally:
  - $intentionally = \lambda x : human.\lambda P : [\![human]\!] .\lambda P{:}Prop.\ I_x(P(x)) \wedge \Gamma(P(x))$

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Adverbial Modification: Intensional/domain adverbials

- Use of TT contexts in this case as well
  - $allegedly = \lambda P : Prop.\ \exists p{:}human, A_p(P)$
  - Someone has alleged that P (A_p is an agent's allegation context [Chatzikyriakidis and Luo(2017b)]
- Introduction of intenTional contexts: Contexts including the intentions (rather than the beliefs) of an agent. We can use this idea for adverbs like intentionally:
  - $intentionally = \lambda x : human.\lambda P : [\![human]\!] .\lambda P{:}Prop.\ I_x(P(x)) \wedge \Gamma(P(x))$
- Domain adverbs, e.g. *botanically, mathematically*
  - $botanically = \lambda P : Prop.\Gamma_B P$
- Intensionality without possible worlds

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Co-predication and Dot-types

- Predicates requiring different kinds of arguments, are used in coordination and applied to the "same" CN argument

Co-predication and Dot-types

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(2) John picked up and mastered the book.

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Co-predication and Dot-types

- Predicates requiring different kinds of arguments, are used in coordination and applied to the "same" CN argument

  (3) John picked up and mastered the book.

- A physical and an informational dimension of *book*

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Co-predication and Dot-types

- Predicates requiring different kinds of arguments, are used in coordination and applied to the "same" CN argument

  (4) John picked up and mastered the book.

- A physical and an informational dimension of *book*
  - The idea is that book is a complex type with both a physical and an informational aspect

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Co-predication and Dot-types

- Predicates requiring different kinds of arguments, are used in coordination and applied to the "same" CN argument

  (5) John picked up and mastered the book.

- A physical and an informational dimension of *book*
  - The idea is that book is a complex type with both a physical and an informational aspect
    - ⋆ We introduce the dot type constructor, forming complex types from simple types
    - ⋆ To form a dot type $A \bullet B$, its individual components should not share parts
    - ⋆ E.g. $\textsc{Phy} \bullet \textsc{Phy}$ cannot be a dot-type
    - ⋆ The dot-type is a subtype of its individual parts

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Co-predication and Dot-types

- Predicates requiring different kinds of arguments, are used in coordination and applied to the "same" CN argument

(6) John picked up and mastered the book.

- A physical and an informational dimension of *book*
  - The idea is that book is a complex type with both a physical and an informational aspect
    - We introduce the dot type constructor, forming complex types from simple types
    - To form a dot type $A \bullet B$, its individual components should not share parts
    - E.g. $\textsc{Phy} \bullet \textsc{Phy}$ cannot be a dot-type
    - The dot-type is a subtype of its individual parts
  - A scary slide follows!

The rules for dot-types

Formation Rule

$$\frac{\Gamma\ valid \quad \langle\rangle \vdash A : Type \quad \langle\rangle \vdash B : Type \quad C(A) \cap C(B) = \emptyset}{\Gamma \vdash A \bullet B : Type}$$

Introduction Rule

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B \quad \Gamma \vdash A \bullet B : Type}{\Gamma \vdash \langle a, b\rangle : A \bullet B}$$

Elimination Rules

$$\frac{\Gamma \vdash c : A \bullet B}{\Gamma \vdash p_1(c) : A} \qquad \frac{\Gamma \vdash c : A \bullet B}{\Gamma \vdash p_2(c) : B}$$

Computation Rules

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B \quad \Gamma \vdash A \bullet B : Type}{\Gamma \vdash p_1(\langle a, b\rangle) = a : A} \qquad \frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B \quad \Gamma \vdash A \bullet B : Typ}{\Gamma \vdash p_2(\langle a, b\rangle) = b : B}$$

Projections as Coercions

$$\frac{\Gamma \vdash A \bullet B : Type}{\Gamma \vdash A \bullet B <_{p_1} A : Type} \qquad \frac{\Gamma \vdash A \bullet B : Type}{\Gamma \vdash A \bullet B <_{p_2} B : Type}$$

Coercion Propagation

$$\frac{\Gamma \vdash A \bullet B : Type \quad \Gamma \vdash A' \bullet B' : Type \quad \Gamma \vdash A <_{c_1} A' : Type \quad \Gamma \vdash B = B' : Type}{\Gamma \vdash A \bullet B <_{d_1[c_1]} A' \bullet B' : Type}$$

where $d_1[c_1](x) = \langle c_1(p_1(x)), p_2(x)\rangle$.

$$\frac{\Gamma \vdash A \bullet B : Type \quad \Gamma \vdash A' \bullet B' : Type \quad \Gamma \vdash A = A' : Type \quad \Gamma \vdash B <_{c_2} B' : Type}{\Gamma \vdash A \bullet B <_{d_2[c_2]} A' \bullet B' : Type}$$

where $d_2[c_2](x) = \langle p_1(x), c_2(p_2(x))\rangle$.

$$\frac{\Gamma \vdash A \bullet B : Type \quad \Gamma \vdash A' \bullet B' : Type \quad \Gamma \vdash A <_{c_1} A' : Type \quad \Gamma \vdash B <_{c_2} B' : Type}{\Gamma \vdash A \bullet B <_{d[c_1,c_2]} A' \bullet B' : Type}$$

where $d[c_1, c_2](x) = \langle c_1(p_1(x)), c_2(p_2(x))\rangle$.

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Dot-types

$John : Man$

$picked\_up : phy \rightarrow human \rightarrow Prop$

$mastered : info \rightarrow human \rightarrow Prop$

$the : \Pi A : CN.A$

$book : CN$

$and : \Pi A.A \rightarrow A \rightarrow A$

Works because $Book \leq Phy \bullet Info \leq Info, Phy$

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Dot-types

$John{:}Man$

$picked\_up{:}phy \rightarrow human \rightarrow Prop$

$mastered{:}info \rightarrow human \rightarrow Prop$

$the{:}\Pi A{:}CN.A$

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- Individuation criteria: John picked up and mastered three books

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- Individuation criteria: John picked up and mastered three books
  - Every CN carries its own criteria of identity: CNs as setoids

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Criteria of Identity/Individuation: CNs as Setoids

- Individuation is the process by which objects in a particular collection are distinguished from one another
  - Provides us with means to count and a sameness criterion
- In linguistic semantics, individuation is related to the idea that a CN may have its own identity criterion for individuation [Geach(1962)]
- Mathematically, the association of an equivalence relation (the identity criterion) CNs
  - In constructive mathematics, a set or a type is indeed a collection of objects together with an equivalence relation that serves as identity criterion of that collection

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CNs as Setoids

- CNs are not just types
  - Types plus an identity criterion for that specific CN

    (7) $(A, =)$

    where A is a type and $=: A \rightarrow A \rightarrow Prop$ is an equivalence relation over A
  - The difference between CNs-as-Types and CNs-as-Setoids

    (8) $[human] = Human : Type$ (CNs-as-types view)

    (9) $[human] = (Human, =_h)$ (CNs-as-Setoids view)

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CNs as Setoids

- Consider the following examples and their semantic interpretations:

(10) Three men talk.

(11) Three humans talk.

(12) $\exists x, y, z : Man.\ x \neq_M y \ \&\ y \neq_M z \ \&\ x \neq_M z \ \&\ talk(x) \& talk(y) \& talk(z)$

(13) $\exists x, y, z : Human.\ x \neq_H y \ \&\ y \neq_H z \ \&\ x \neq_H z \ \&\ talk(x) \& talk(y) \& talk(z)$

where $\textsc{Man} = (Man, =_M)$ and $\textsc{Human} = (Human, =_H)$ are setoids and the identity criterion for men and that for humans are used to express that x, y and z are distinct from each other.

CNs as Setoids

- Necessary to consider the individuation criteria explicitly by using the identity criteria $=_M$ and $=_H$
- The relationship between the $\textsc{Man}$ and $\textsc{Human}$ is one where the first inherits the IC from the second

(14) $(=_M) = (=_H)|_{Man}$

CNs as Setoids: Subsetoids

- $\mathrm{A} = (A, =_\mathrm{A})$ is a sub-setoid of $\mathrm{B} = (B, =_{\mathrm{B})}$, notation $\mathrm{A} \sqsubseteq \mathrm{B}$, iff
  - $A \leq B$ and $=_\mathrm{A}$ is the same as $(=_\mathrm{B})|_A$ (the restriction of $=_\mathrm{B}$ over A).
- Some examples:

(15) $\textsc{Man} \sqsubseteq \textsc{Human}$

(16) $(RTable, =_t) \sqsubseteq (Table, = t)$
where $RTable$ is: $\Sigma x{:}Table.red(x)$ is the domain of red tables and $=_t$ is the equivalence relation representing the identity criterion for tables

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CNs as Setoids: Subsetoids

- In restricted domains like *Man* or *RTable*, the identity criteria coincide with those in *Human* and *Table*
- For these cases, one can ignore the IC, i.e. one can use the simpler CNs-as-Types approach
  - More sophisticated cases like copredication with quantification however need IC

(17) John picked up and mastered three books.

- Double distinctness

(18) John picked up and mastered three books $\Rightarrow$ John picked up three physical objects and mastered three informational objects

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CNs as Setoids: Copredication

- Let us split the example into its conjuncts

(19) $Three(Book, \textsc{Phy}, pick\ up(j))$.

(20) $Three(Book, \textsc{Info}, master(j))$.

- Note: the CN *book* in 19 refers to a different collection from that referred to by *book* in 20

(21) $\textsc{Book}_1 = (Book, =_p)$

(22) $\textsc{Book}_2 = (Book, =_i)$

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CNs as Setoids: Copredication

- How the identity criterion for *books* is determined
  - why do we use $=_p$ in 19 and $=_i$ in 20?
    - ★ The verb (and its semantics) that determines the identity criterion of the object CN.

$$IC^{N,V} \Rightarrow \begin{cases} =_p & \text{if Dom(V)} = \textsc{Phy} \\ =_i & \text{if Dom(V)} = \textsc{Info} \\ ??? & \text{if Dom(V)} = \textsc{Phy} \bullet \textsc{Info} \end{cases}$$

    - ★ In order to deal with the dot-type case, we have to define setoids for dot-types!

CNs as Setoids: Copredication

- Let $\mathrm{A} = (A, =_{\mathrm{A}})$ and $\mathrm{B} = (B, =_{\mathrm{B}})$ be setoids. Then, the *dot-setoid* $\mathrm{A} \bullet \mathrm{B}$ is defined as follows:
  - $\mathrm{A} \bullet \mathrm{B} = (A \bullet B,\ =_{\mathrm{A} \bullet \mathrm{B}})$
    where $\langle a_1, b_1 \rangle =_{\mathrm{A} \bullet \mathrm{B}} \langle a_2, b_2 \rangle$ if, and only if, $(a_1 =_{\mathrm{A} a_2}) \vee (b_1 =_{\mathrm{B}} b_2)$.

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CNs as Setoids: Copredication

- The semantics for *three*
  Let A be a type and $\mathrm{B} = (B, =_{\mathrm{B}})$ a setoid such that $A \leq B$, and $P : B \rightarrow Prop$ a predicate over B:
- $Three(A, \mathrm{B}, P) = \exists x, y, z : A.\ D[\mathrm{B}](x, y, z)\ \&\ P(x)\ \&\ P(y)\ \&\ P(z)$.
  where $D[\mathrm{B}](x, y, z) = x \neq_{\mathrm{B}} y\ \&\ y \neq_{\mathrm{B}} z\ \&\ x \neq_{\mathrm{B}} z$.

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CNs as Setoids: Copredication

- With these definitions, the desired semantics of our copredication cases are derived
- $\mathit{Three}(\mathit{Book}, \textsc{Phy} \bullet \textsc{Info}, \mathit{pm}(j))$
- $\exists x, y, z : \mathit{Book}.D[\textsc{Phy}](x, y, z)$ & $D[\textsc{Info}](x, y, z)$ & $\mathit{pm}(j, x)$ & $\mathit{pm}(j, y)$ & $\mathit{pm}(j, z)$
    - Note that this is achieved through defining the equivalence relation for dot-types by means of disjunction of both identity criteria and, then, we obtain double distinctness by negating the disjunction.

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CNs as Setoids: Copredication

- Verbs plus adjectives in quantified copredication
  - Consider the following example:

    (23) John mastered three heavy books.

  - The interpretation needed here: John mastered three informational objects that are also heavy as physical objects
    - ★ Both the verb and the adjective have a word on the IC
- First step: adjectival modification
  - $HBook = \Sigma(Book, heavy)$ or $\Sigma x{:}Book.heavy(x)$

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CNs as Setoids: Copredication

- The interpretation we get:

  $Three(HBook, \textsc{Phy} \bullet \textsc{Info}, master(j))$

- Expanding:

  $\exists x, y, z : HBook.D[\textsc{Phy}](x, y, z) \ \& \ D[\textsc{Info}](x, y, z)$
  $\& \ master(j, x) \ \& \ master(j, y) \ \& \ master(j, z)$

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Conclusions

- MTTs as foundational languages for formal semantics

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  - Formally, well-studied

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Conclusions

- MTTs as foundational languages for formal semantics
  - Formally, well-studied
  - Expressively adequate
  - Proof-theoretically specified, supporting effecting reasoning
- State of maturity of both MTT semantics and proof assistant technology
  - Use proof assistant technology and MTTs for formal verification and inference

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