Introduction to Dependent Type Semantics

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Dependent Type Semantics (DTS) (Bekki 2014; Bekki and Mineshima 2017)

- A framework of natural language semantics
- Unified approach to general inferences and anaphora/presupposition resolution in terms of *proof construction* (cf. Krahmer and Piwek (1999))

Main features:

1. **Proof-theoretic semantics**:
   From truth-conditions (denotations, models) to proof-conditions (proofs, contexts)
2. **Underspecification Semantics**: A proof-theoretic alternative to Dynamic Semantics (DRT, DPL, etc.)
3. **Compositional semantics**: Syntax-semantics interface via categorial grammars (e.g. CCG, Hybrid-TLCG)
4. **Computational semantics**: Implementation, Applications to Natural Language Processing

Dependent Types

Per Martin-Löf

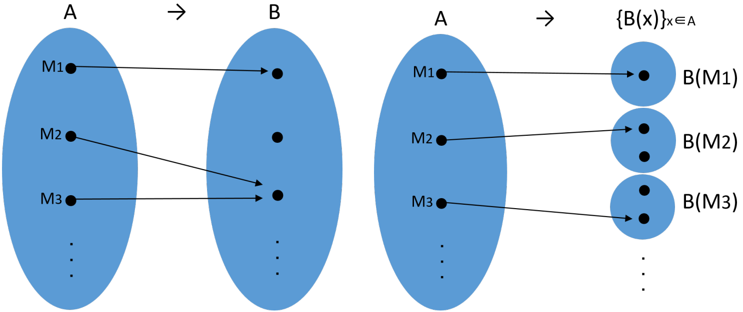
Martin-Löf (1984) "Intuitionistic type theory"

What are Π-types

Π-type is a type of *fibred* functions.

Simple function space

Fibred function space

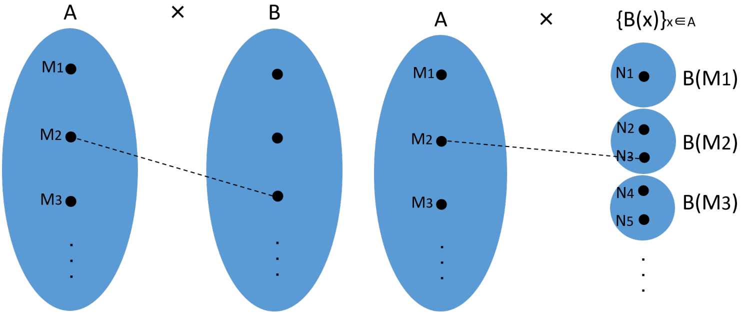


What are Σ-types

Σ-type is a type of *fibred* products.

Simple product space

Fibred product space



Notations

DTS notation	Standard notation	$x \notin fv(B)$	$x \in fv(B)$
$(x : A) \rightarrow B$	$(\Pi x : A)B$	$A \rightarrow B$	$(\forall x : A)B$
$(x : A) \times B$ *or* $\begin{bmatrix} x{:}A \\ B \end{bmatrix}$	$(\Sigma x : A)B$	$A \wedge B$	$(\exists x : A)B$

Scope of the variable in Π-types: $(x : A) \rightarrow B$

Scope of the variable in Σ-types: $\begin{bmatrix} x{:}A \\ B \end{bmatrix}$

Π-type F/I/E rules

$$\dfrac{A : \mathsf{type}_i \quad \begin{array}{c}\overline{x : A}^{\,k} \\ \vdots \\ B : \mathsf{type}_j\end{array}}{(x : A) \to B : \mathsf{type}_{max(i,j)}}\,(\Pi F),k$$

$$\dfrac{A : \mathsf{type}_i \quad \begin{array}{c}\overline{x : A}^{\,k} \\ \vdots \\ M : B\end{array}}{\lambda x.M : (x : A) \to B}\,(\Pi I),k \qquad \dfrac{M : (x : A) \to B \quad N : A}{MN : B[N/x]}\,(\Pi E)$$

Σ-type F/I/E rules

$$\dfrac{A : \mathsf{type}_i \quad \begin{array}{c}\overline{x : A}^{\,k} \\ \vdots \\ B : \mathsf{type}_j\end{array}}{(x : A) \times B : \mathsf{type}_{max(i,j)}} \; (\Sigma F),k$$

$$\dfrac{M : A \quad N : B[M/x]}{(M, N) : (x : A) \times B} \; (\Sigma I)$$

$$\dfrac{M : (x : A) \times B}{\pi_1(M) : A} \; (\Sigma E) \qquad \dfrac{M : (x : A) \times B}{\pi_2(M) : B[\pi_1(M)/x]} \; (\Sigma E)$$

Dependent Types prevail

Dependent Types prevail

Rules of DTS

Rules from Martin-Löf Type Theory

- Axioms and Structural rules
- $\mathbf{\Pi}$-type (Dependent function type) [F/I/E]
- $\mathbf{\Sigma}$-type (Dependent product type) [F/I/E]
- Intensional equality type [F/I/E]
- Disjoint union type [F/I/E]
- Enumeration type [F/I/E]
- Natural number type [F/I/E]

New rule in DTS

- @ (the ‘asperand’ operator)
  - Anaphora and presupposition triggers (linguistically speaking)
  - Open proofs (logically speaking)

Conjunction, Implication, and Negation

Definition

$$\begin{bmatrix} A \\ B \end{bmatrix} \stackrel{def}{\equiv} (x : A) \times B \qquad where\ x \notin fv\,(B)$$

$$A \rightarrow B \stackrel{def}{\equiv} (x : A) \rightarrow B \qquad where\ x \notin fv\,(B)$$

$$\neg A \stackrel{def}{\equiv} (x : A) \rightarrow \bot$$

Dynamics in Natural Language Semantics

A theory of anaphora

- Anaphora representable by a constant symbol:
  - Deictic use:

    (1) *(Pointing at John)*

    He was born in Detroit.

    $\mathbf{bornIn}(j, d)$

  - Coreference:

    (2) John loves a girl who hates him.

    $\exists x(\mathbf{girl}(x) \wedge \mathbf{love}(j, x) \wedge \mathbf{hate}(x, j))$

- Anaphora representable by a variable
  - Bound variable anaphora:

    (3) Every boy loves his father.

    $\forall x(\mathbf{boy}(x) \rightarrow \mathbf{love}(x, \mathbf{fatherOf}(x)))$

A theory of anaphora

- Anaphora not representable by FoL:
  - E-type anaphora:

    (4) A man entered into the park. He whistled.

  - Donkey anaphora:

    (5) Every farmer who owns a donkey beats it.

    (6) If a farmer owns a donkey, he beats it.

- Anaphora not representable by FoL nor dynamic semantics:
  - Syllogistic anaphora:

    (7) Every girl received a present. Some girl opened it.

  - Disjunctive antecedent:

    (8) If Mary sees a horse or a pony, she waves to it.

E-type anaphora: Evans (1980)

(9) [A man]^1 entered. He$_1$ whistled.

The first-order SR (10) represents the truth condition of (9), thus is a candidate of the SR of (9).

(10) $\exists x(\mathbf{man}(x) \wedge \mathbf{enter}(x) \wedge \mathbf{whistle}(x))$

But the syntactic structure of the SR (10) does not correspond to that of (9), where consists of two independent sentences. The sentential boundary of (9) prefers the first-order representation (11).

(11) $\exists x(\mathbf{man}(x) \wedge \mathbf{enter}(x)) \wedge \mathbf{whistle}(x)$

However, the truth condition of (11) is different from that of the mini-discourse (9) since the variable x in $\mathbf{whistle}(x)$ is not bound by $\exists$.

Donkey anaphora: Geach (1962)

For the donkey sentences (12), a first-order formula (13), whose truth condition is the same as those of (12), is a candidate of its SR.

(12) a. Every farmer who owns [a donkey]^{1} beats it$_{1}$.

b. If [a farmer]^{1} owns [a donkey]^{2}, he$_{1}$ beats it$_{2}$.

(13) $\forall x(\mathbf{farmer}(x) \to \forall y(\mathbf{donkey}(y) \land \mathbf{own}(x, y) \to \mathbf{beat}(x, y)))$

But the translation from the sentence (12) to (13) is not straightforward since i) the indefinite noun phrase *a donkey* is translated into a universal quantifier in (13) instead of an existential quantifier, and ii) the syntactic structure of (13) does not corresponds to that of (12).

Donkey anaphora: Geach (1962)

(12) a. Every farmer who owns [a donkey]$^{\mathbf{1}}$ beats it$_{\mathbf{1}}$.

b. If [a farmer]$^{\mathbf{1}}$ owns [a donkey]$^{\mathbf{2}}$, he$_{\mathbf{1}}$ beats it$_{\mathbf{2}}$.

The syntactic parallel of (12) is, rather, the SR (14), in which the indefinite noun phrase is translated into an existential quantification.

(14) $\forall x(\mathbf{farmer}(x) \wedge \exists y(\mathbf{donkey}(y) \wedge \mathbf{own}(x, y)) \longrightarrow \mathbf{beat}(x, \boldsymbol{y}))$

However, (14) does not represent the truth condition of (12) correctly since the variable $\boldsymbol{y}$ in $\mathbf{beat}(\boldsymbol{x}, \boldsymbol{y})$ fails to be bound by $\exists$. Therefore, neither (13) nor (14) qualifies as the SR of (12).

E-type anaphora: Ranta (1994)

(9) A man entered. He whistled.

$$\begin{bmatrix} u : \begin{bmatrix} x : \textbf{entity} \\ \begin{bmatrix} \textbf{man}(x) \\ \textbf{enter}(x) \end{bmatrix} \end{bmatrix} \\ \textbf{whistle}(\pi_1(u)) \end{bmatrix}$$

Note: $\begin{bmatrix} x : A \\ B \end{bmatrix}$ is a type for pairs of A and $B[x]$.

Donkey anaphora: Sundholm (1986)

(12a) Every farmer who owns a donkey beats it .

$$\left(u : \begin{bmatrix} x\text{:entity} \\ \begin{bmatrix} \text{farmer}(x) \\ \begin{bmatrix} v\text{:} \begin{bmatrix} y\text{:entity} \\ \text{donkey}(y) \end{bmatrix} \\ \text{own}(x, \pi_1 v) \end{bmatrix} \end{bmatrix} \end{bmatrix} \right) \rightarrow \text{beat}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u)$$

Note: $(x : A) \rightarrow B$ is a type for functions from A to $B[x]$.

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: How could we lexicalize context-dependent words like pronouns?

$$\left[\begin{array}{l} u : \left[\begin{array}{l} x : \textbf{entity} \\ \left[\begin{array}{l} \textbf{man}(x) \\ \textbf{enter}(x) \end{array}\right] \end{array}\right] \\ \textbf{whistle}(\pi_1(u)) \end{array}\right]$$

Dependent Type Semantics (DTS)

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: How could we lexicalize context-dependent words like pronouns?

A: By using **underspecified terms**.

Q: But how could we retrieve a context for an underspecified term?

A: By **type checking**.

Underspecified terms

DTS $=$ DTT $+$ underspecified terms $@_i A$

Definition (@-rule)

$$\frac{A : \text{type} \quad A \ \textit{true}}{@_i A : A} (@)$$

- @-rule states that the well-formedness of $@_i A$ requires:
  - A is a well-formed type.
  - the inhabitance of A (namely, A is a presuppositional content)
- i is an index that distinguishes different underspecified terms

On parsing

We assume that an SR of a sentence is obtained by parsing and semantic composition (assumed by one's syntactic theory).

A sentence

$\Downarrow$

Parsing

$\Downarrow$

An underspecified SRs in DTS

Lexical items in CCG-style

PF	CCG categories	Semantic representations in DTS
if	$S/S/S$	$\lambda p.\lambda q.\,(u:p)\to q$
every$_{nom}$	$S/(S\backslash NP)/N$	$\lambda n.\lambda p.\left(u:\begin{bmatrix} x{:}\mathsf{entity} \\ nx \end{bmatrix}\right)\to p(\pi_1(u))$
every$_{acc}$	$T\backslash(T/NP)/N$	$\lambda n.\lambda p.\lambda\vec{x}.\left(v:\begin{bmatrix} y{:}\mathsf{entity} \\ ny \end{bmatrix}\right)\to p(\pi_1(v))\vec{x}$
a$_{nom}$, some$_{nom}$	$S/(S\backslash NP)/N$	$\lambda n.\lambda p.\begin{bmatrix} u{:}\begin{bmatrix} x{:}\mathsf{entity} \\ nx \end{bmatrix} \\ p(\pi_1(u)) \end{bmatrix}$
a$_{acc}$, some$_{acc}$	$T\backslash(T/NP)/N$	$\lambda n.\lambda p.\lambda\vec{x}.\begin{bmatrix} v{:}\begin{bmatrix} y{:}\mathsf{entity} \\ ny \end{bmatrix} \\ p(\pi_1(v))\vec{x} \end{bmatrix}$
farmer	N	**farmer**
donkey	N	**donkey**
who	$N\backslash N/(S\backslash NP)$	$\lambda p.\lambda n.\lambda x.\begin{bmatrix} nx \\ px \end{bmatrix}$
whom	$N\backslash N/(S/NP)$	$\lambda p.\lambda n.\lambda x.\begin{bmatrix} nx \\ px \end{bmatrix}$
owns	$S\backslash NP/NP$	**own**
beats	$S\backslash NP/NP$	**beat**
he$_i$	NP	$\pi_1\left(@_i\begin{bmatrix} x{:}\mathsf{entity} \\ \mathsf{male}(x) \end{bmatrix}\right)$
it$_i$	NP	$\pi_1\left(@_i\begin{bmatrix} x{:}\mathsf{entity} \\ \neg\mathsf{human}(x) \end{bmatrix}\right)$
the$_i$	NP/N	$\lambda n.\pi_1\left(@_i\begin{bmatrix} x{:}\mathsf{entity} \\ nx \end{bmatrix}\right)$

Lexical items in CCG-style (anaphoric expressions)

PF	CCG categories	Semantic representations in DTS
he$_i$	$\boldsymbol{NP}$	$\boldsymbol{\pi_1}\left(@_i \begin{bmatrix} x\text{:}\textbf{entity} \\ \textbf{male}(x) \end{bmatrix}\right)$
it$_i$	$\boldsymbol{NP}$	$\boldsymbol{\pi_1}\left(@_i \begin{bmatrix} x\text{:}\textbf{entity} \\ \neg\textbf{human}(x) \end{bmatrix}\right)$
the$_i$	$\boldsymbol{NP/N}$	$\boldsymbol{\lambda n.\pi_1}\left(@_i \begin{bmatrix} x\text{:}\textbf{entity} \\ \boldsymbol{nx} \end{bmatrix}\right)$

E-type anaphora: Parsing

$$\dfrac{\dfrac{\dfrac{\text{A}}{\begin{matrix} S/(S\backslash NP)/N \\ \lambda n.\lambda p.\begin{bmatrix} u:\begin{bmatrix} x{:}\mathsf{entity} \\ nx \end{bmatrix} \\ p(\pi_1 u) \end{bmatrix}\end{matrix}} \quad \dfrac{\text{man}}{\begin{matrix} N \\ \lambda x.\mathsf{man}(x) \end{matrix}}}{\begin{matrix} S/(S\backslash NP) \\ \lambda p.\begin{bmatrix} u:\begin{bmatrix} x{:}\mathsf{entity} \\ \mathsf{man}(x) \end{bmatrix} \\ p(\pi_1 u) \end{bmatrix}\end{matrix}}> \quad \dfrac{\text{entered}}{\begin{matrix} S\backslash NP \\ \lambda x.\mathsf{enter}(x) \end{matrix}}}{\begin{matrix} S \\ \begin{bmatrix} u:\begin{bmatrix} x{:}\mathsf{entity} \\ \mathsf{man}(x) \end{bmatrix} \\ \mathsf{enter}(\pi_1 u) \end{bmatrix}\end{matrix}}>$$

E-type anaphora: Parsing

$$\dfrac{\dfrac{\text{He}}{\begin{array}{c} \boldsymbol{NP} \\ \pi_1 \; @_1 \begin{bmatrix} x{:}\mathbf{entity} \\ \mathbf{male}(x) \end{bmatrix} \end{array}} \qquad \dfrac{\text{whistled}}{\begin{array}{c} \boldsymbol{S \backslash NP} \\ \lambda x.\mathbf{whistle}(x) \end{array}}}{\begin{array}{c} \boldsymbol{S} \\ \mathbf{whistle}\left(\pi_1 \; @_1 \begin{bmatrix} x{:}\mathbf{entity} \\ \mathbf{male}(x) \end{bmatrix}\right) \end{array}} <$$

Progressive conjunction: Ranta (1994)

Definition (Progressive conjunction)

$$M;N \stackrel{def}{\equiv} \begin{bmatrix} u{:}M \\ N \end{bmatrix} \qquad \text{where } u \notin fv(N)$$

(9) [A man][1] entered. He $_1$ whistled.

$$\begin{bmatrix} x{:}\mathsf{entity} \\ \begin{bmatrix} \mathsf{man}(x) \\ \mathsf{enter}(x) \end{bmatrix} \end{bmatrix} ; \ \mathsf{whistle}(\pi_1 \ @_1 \begin{bmatrix} x{:}\mathsf{entity} \\ \mathsf{male}(x) \end{bmatrix})$$

$$= \begin{bmatrix} u{:} \begin{bmatrix} x{:}\mathsf{entity} \\ \begin{bmatrix} \mathsf{man}(x) \\ \mathsf{enter}(x) \end{bmatrix} \end{bmatrix} \\ \mathsf{whistle}(\pi_1 \ @_1 \begin{bmatrix} x{:}\mathsf{entity} \\ \mathsf{male}(x) \end{bmatrix}) \end{bmatrix}$$

E-type anahora: Type checking

$$
\dfrac{
\begin{array}{c}\vdots\\ \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix} : \text{type}\end{array}
\qquad
\begin{array}{c}
\overline{\begin{bmatrix} u\text{:}\begin{bmatrix} x\text{:entity} \\ \text{man}(x) \end{bmatrix} \\ \text{enter}(\pi_1 u) \end{bmatrix}}^{\,1} \\ \vdots \\ \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix} \ \textit{true}
\end{array}
}{
@_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix} : \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix}
}\ (@)
$$

$$
\dfrac{@_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix} : \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix}}{\pi_1\ @_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix} : \text{entity}}\ (\Sigma E)
$$

$$
\dfrac{}{\text{whistle} : \text{entity} \longrightarrow \text{type}}\ (CON)
$$

$$
\dfrac{\text{whistle} : \text{entity} \longrightarrow \text{type} \qquad \pi_1\ @_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix} : \text{entity}}{\text{whistle}\left(\pi_1\ @_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix}\right) : \text{type}}\ (\to E)
$$

$$
\dfrac{
\begin{array}{c}\vdots\\ \begin{bmatrix} u\text{:}\begin{bmatrix} x\text{:entity} \\ \text{man}(x) \end{bmatrix} \\ \text{enter}(\pi_1 u) \end{bmatrix} : \text{type}\end{array}
\qquad
\text{whistle}\left(\pi_1\ @_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix}\right) : \text{type}
}{
\begin{bmatrix} v\text{:}\begin{bmatrix} u\text{:}\begin{bmatrix} x\text{:entity} \\ \text{man}(x) \end{bmatrix} \\ \text{enter}(\pi_1 u) \end{bmatrix} \\ \text{whistle}\left(\pi_1\ @_1 \begin{bmatrix} x\text{:entity} \\ \text{male}(x) \end{bmatrix}\right) \end{bmatrix} : \text{type}
}\ (\Sigma F), 1
$$

E-type anaphora: Proof search

$$
\dfrac{
\dfrac{
\dfrac{
\overline{v : \begin{bmatrix} u : \begin{bmatrix} x:\text{entity} \\ \text{man}(x) \end{bmatrix} \\ \text{enter}(\pi_1 u) \end{bmatrix}}^{\,1}
}{
\pi_1 v : \begin{bmatrix} x:\text{entity} \\ \text{man}(x) \end{bmatrix}
}(\Sigma E)
}{
\pi_1\pi_1 v : \text{entity}
}(\Sigma E)
\qquad
\dfrac{
\dfrac{
\overline{v : \begin{bmatrix} u : \begin{bmatrix} x:\text{entity} \\ \text{man}(x) \end{bmatrix} \\ \text{enter}(\pi_1 u) \end{bmatrix}}^{\,1}
}{
\pi_1 v : \begin{bmatrix} x:\text{entity} \\ \text{man}(x) \end{bmatrix}
}(\Sigma E)
\quad
\overline{m : \left(u : \begin{bmatrix} x:\text{entity} \\ \text{man}(x) \end{bmatrix} \right) \to \text{male}(\pi_1 u)}\,(CON)
}{
m(\pi_1 v) : \text{male}(\pi_1\pi_1 v)
}(\Pi E)
}{
(\pi_1\pi_1 v, m(\pi_1 v)) : \begin{bmatrix} x:\text{entity} \\ \text{male}(x) \end{bmatrix}
}(\Sigma E)
$$

E-type anaphora: @-elimination

$$
\dfrac{
\begin{matrix}\vdots\\ \begin{bmatrix} u{:}\begin{bmatrix}x{:}\mathsf{entity}\\ \mathsf{man}(x)\end{bmatrix}\\ \mathsf{enter}(\pi_1 u)\end{bmatrix} : \mathsf{type}\end{matrix}
\qquad
\dfrac{\mathsf{whistle} : \mathsf{entity} \longrightarrow \mathsf{type} \qquad \dfrac{\begin{matrix}\overline{v : \begin{bmatrix} u{:}\begin{bmatrix}x{:}\mathsf{entity}\\ \mathsf{man}(x)\end{bmatrix}\\ \mathsf{enter}(\pi_1 u)\end{bmatrix}}^{\,1}\\ \vdots \\ (\pi_1\pi_1 v, m(\pi_1 v)) : \begin{bmatrix}x{:}\mathsf{entity}\\ \mathsf{male}(x)\end{bmatrix}\end{matrix}}{\pi_1\pi_1 v : \mathsf{entity}}(\Sigma E)}{\mathsf{whistle}(\pi_1\pi_1 v) : \mathsf{type}}(\Pi E)
}{
\begin{bmatrix} v{:}\begin{bmatrix} u{:}\begin{bmatrix}x{:}\mathsf{entity}\\ \mathsf{man}(x)\end{bmatrix}\\ \mathsf{enter}(\pi_1 u)\end{bmatrix}\\ \mathsf{whistle}(\pi_1\pi_1 v)\end{bmatrix} : \mathsf{type}
}(\Sigma F),1
$$

@-elimination rules

Definition (@-elimination rules (excerpt))

$$\left[\!\!\left[\dfrac{\begin{array}{cc} & \mathcal{D}_M \\ A : \mathsf{s} & M : A' \end{array}}{@_i A : A}\,(@)\right]\!\!\right] = \begin{array}{c}\mathcal{D}_M \\ M : A'\end{array}$$

$$\left[\!\!\left[\dfrac{\begin{array}{cc} & x : A' \\ \mathcal{D}_A & \mathcal{D}_B \\ A : \mathsf{s}_1 & B : \mathsf{s}_2 \end{array}}{(x : A) \to B : \mathsf{s}_2}\,(\underline{\Pi F})\right]\!\!\right] = \dfrac{\begin{array}{cc} & x : A' \\ [\![\mathcal{D}_A]\!] & [\![\mathcal{D}_B]\!] \\ A' : \mathsf{s}_1 & B' : \mathsf{s}_2 \end{array}}{(x : A') \to B' : \mathsf{s}_2}\,(\Pi F)$$

$$\left[\!\!\left[\dfrac{\begin{array}{cc} & x : A' \\ \mathcal{D}_A & \mathcal{D}_M \\ A : \mathsf{s}_1 & M : B \end{array}}{\lambda x.M : (x : A) \to B}\,(\underline{\Pi I})\right]\!\!\right] = \dfrac{\begin{array}{cc} & x : A' \\ [\![\mathcal{D}_A]\!] & [\![\mathcal{D}_M]\!] \\ A' : \mathsf{s}_1 & M' : B' \end{array}}{\lambda x.M' : (x : A') \to B'}\,(\Pi I)$$

$$\left[\!\!\left[\dfrac{\begin{array}{cc} \mathcal{D}_M & \mathcal{D}_N \\ M : (x : A) \to B & N : A \end{array}}{MN : B'}\,(\underline{\Pi E})\right]\!\!\right] = \dfrac{\begin{array}{cc} [\![\mathcal{D}_M]\!] & [\![\mathcal{D}_N]\!] \\ M' : (x : A') \to B' & N' : A' \end{array}}{M'N' : B''}\,(\Pi E)$$

where $B'[N'/x] \twoheadrightarrow_\beta B''$

The model of language understanding

A sentence ... A sentence

⇓ ⇓

Parsing ... Parsing

⇓ ⇓

An underspecified SRs in DTS ... An underspecified SRs in DTS

⇓ ⇓

Discoruse relation (e.g. Progressive conjunction)

⇓

An underspecified discourse representation in DTS

⇓

Type checking + Proof search in DTS

⇓

A proof diagram of the well-formedness of an SR in DTS

⇓

@-elimination

⇓

A proof diagram of the well-formedness of an SR in DTT

⇓

Inference in DTT

Beyond Truth-conditions

Anaphoric potential

The sentences (15a) and (16a) are truth-conditionally equivalent (cf. $\exists x(\mathbf{L}x \land \mathbf{R}x) \equiv \neg\forall x(\mathbf{L}x \rightarrow \neg\mathbf{R}x)$), while they show the different *anaphoric potential* to the subsequent sentences.

(15) a. Some linguist is rich.

b. He/She does not even have to teach.

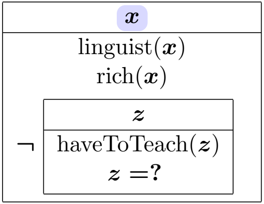
(16) a. It is not the case that no linguist is rich.

b. * He/She does not even have to teach.

Therefore, truth-conditinal semantics is not enough for distinguishing a certain aspect of the meaning of sentences. (Kamp et al. (2011))

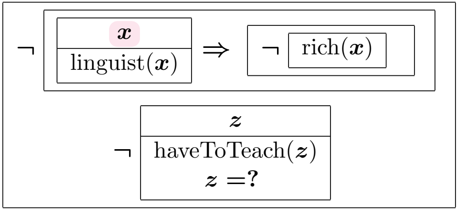
Anaphoric potential in DRT

Some linguist is rich.
He/she does not even have to teach.



x is accessible from z.

It is not the case that no linguist is rich.
**He/she does not even have to teach.*



x is not accessible from z.

Anaphoric potential in DTS

Some linguist is rich.
He/she does not even have to teach.

$$\begin{bmatrix} u : \begin{bmatrix} x:\textsf{entity} \\ \begin{bmatrix} \textsf{L}(x) \\ \textsf{R}(x) \end{bmatrix} \end{bmatrix} \\ \neg \textsf{HTT}(@_1\textsf{entity}) \end{bmatrix}$$

$$@_1\textsf{entity} = \pi_1 u$$

It is not the case that no linguist is rich.
He/she does not even have to teach.

$$\begin{bmatrix} u : \neg\,(x:\textsf{entity}) \to \textsf{L}(x) \to \neg\textsf{R}(x) \\ \neg \textsf{HTT}(@_1\textsf{entity}) \end{bmatrix}$$

$$@_1\textsf{entity} = ??$$

Anaphoric potential in DTS

However, if the following inference had a proof term $f\ u$, then the anaphoric link in question would be wrongly predicted to be licenced.

$$u : \neg(x : \mathsf{entity}) \to \mathsf{L}(x) \to \neg\mathsf{R}(x) \vdash \begin{bmatrix} x{:}\mathsf{entity} \\ \begin{bmatrix} \mathsf{L}(x) \\ \mathsf{R}(x) \end{bmatrix} \end{bmatrix}\ \mathit{true}$$

Proof search:

$$\dfrac{\dfrac{}{\mathsf{entity} : \mathsf{type}}(CON) \qquad \dfrac{\dfrac{u : \neg(x : \mathsf{entity}) \to \mathsf{L}(x) \to \neg\mathsf{R}(x)}{f\ u : \begin{bmatrix} x{:}\mathsf{entity} \\ \begin{bmatrix} \mathsf{L}(x) \\ \mathsf{R}(x) \end{bmatrix} \end{bmatrix}}?}{\pi_1(f\ u) : \mathsf{entity}}(\Sigma E)}{@_1\mathsf{entity} : \mathsf{entity}}(@)$$

Proof of $u : \neg((x : \mathsf{e}) \to \mathsf{L}x \to \neg\mathsf{R}x) \vdash \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix}$ *true*

$$\dfrac{\overline{l : \mathsf{L}x}^{\,2} \qquad \overline{r : \mathsf{R}x}^{\,1}}{(l, r) : \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix}}\,(\Sigma I)$$

$$\dfrac{\overline{x : \mathsf{e}}^{\,3} \qquad (l, r) : \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix}}{(x, (l, r)) : \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix}}\,(\Sigma I)$$

$$\dfrac{(x, (l, r)) : \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix} \qquad \overline{y : \neg \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix}}^{\,4}}{y(x, (l, r)) : \bot}\,(\to E)$$

$$\dfrac{y(x, (l, r)) : \bot}{\lambda r.y(x, (l, r)) : \neg\mathsf{R}x}\,(\neg I),1$$

$$\dfrac{\lambda r.y(x, (l, r)) : \neg\mathsf{R}x}{\lambda l.\lambda r.y(x, (l, r)) : \mathsf{L}x \to \neg\mathsf{R}x}\,(\to I),2$$

$$\dfrac{\lambda l.\lambda r.y(x, (l, r)) : \mathsf{L}x \to \neg\mathsf{R}x}{\lambda x.\lambda l.\lambda r.y(x, (l, r)) : (x : \mathsf{e}) \to \mathsf{L}x \to \neg\mathsf{R}x}\,(\Pi I),3$$

$$\dfrac{u : \neg((x : \mathsf{e}) \to \mathsf{L}x \to \neg\mathsf{R}x) \qquad \lambda x.\lambda l.\lambda r.y(x, (l, r)) : (x : \mathsf{e}) \to \mathsf{L}x \to \neg\mathsf{R}x}{u(\lambda x.\lambda l.\lambda r.y(x, (l, r))) : \bot}\,(\to E)$$

$$\dfrac{u(\lambda x.\lambda l.\lambda r.y(x, (l, r))) : \bot}{\lambda y.u(\lambda x.\lambda l.\lambda r.y(x, (l, r))) : \neg\neg \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix}}\,(\neg I),4$$

$$\dfrac{\lambda y.u(\lambda x.\lambda l.\lambda r.y(x, (l, r))) : \neg\neg \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix}}{\mathsf{dne}(\lambda y.u(\lambda x.\lambda l.\lambda r.y(x, (l, r)))) : \begin{bmatrix} x{:}\mathsf{e} \\ \begin{bmatrix} \mathsf{L}x \\ \mathsf{R}x \end{bmatrix} \end{bmatrix}}\,(DNE)$$

Proof of $u : \neg((x : \textbf{entity}) \to \textbf{L}x \to \neg\textbf{R}x) \vdash \begin{bmatrix} x{:}\textbf{entity} \\ \begin{bmatrix} \textbf{L}x \\ \textbf{R}x \end{bmatrix} \end{bmatrix}$ *true*

- This does not hold in *intuitionistic* settings, such as DTT and DTS.
- It requires *classical* settings, such as DTT/DTS+(DNE)-rule

$$\frac{M : \neg\neg A}{\mathrm{dne}(M) : A}\ (DNE)$$

Proof-theoretic hierarchy

$$\begin{aligned} NK &= NM + \text{DNE (i.e.}\neg\neg A \rightarrow A) \\ NK &= NJ + \text{LEM (i.e.}\neg A \vee A) \\ NJ &= NM + \text{EFQ (i.e.}\bot \rightarrow A) \end{aligned}$$

(15a) Some linguist is rich.

(16a) It is not the case that no linguist is rich.

- The difference in anaphoric potentials of (15a) and (16a) is due to the fact that
  - (15a) $\vdash_{NM}$ (16a)
  - (16a) $\vdash_{NK}$ (15a)

  are theorems of different proof systems.
- Proof-theoretic semantics provides a distinction between truth-conditionally equivalent propositions.

A toilet in a funny place

Similar argument applies to the following case discussed in Karttunen (1976), Krahmer and Muskens (1995), Geurts (1999):

(17) a. Either there is no toilet, or it is in a funny place.

If disjunction in DTT is defined (instead of $\uplus$) as $\boldsymbol{A} \vee \boldsymbol{B} \stackrel{def}{\equiv} \neg \boldsymbol{A} \rightarrow \boldsymbol{B}$, then we can deduce "there is a toilet" from a doubly-negated form "$\neg\neg$ there is a toilet" (Bekki, 2013)

A toilet in a funny place

However, this analysis is not on a right track, since it predicts that the following anaphoric link is licenced if the use of dne is allowed in DTS, which is not the case.

(18) It is not the case that nobody showed up.

* He/she was sitting alone.

Thus we need the other analysis for this case. But the following cases implies that we have to be careful about the status of the original paradigm:

(19) Either John is not married, or *she is not home right now.

(20) It is not the case that there is no toilet in this building.

Actually, it is in a funny place.

Hidden marbles

A well-known example showing that "discourse referents are not introduced solely by inferences". (by Barbara Partee, due to Heim (1982))

(21) Nine balls out of ten are found in the bag. * It must be under the sofa.

We should reconsider this case from a proof-theoretic perspective whether (21) can be proven in a intuitionistic setting.

There are exactly ten balls.
Exactly nine balls are found.

?? There is a ball that is not found.

In this way, DTS provides a new perspective that differentiates the status of various constructions from the viewpoint of proof-theory.

Concluding Remarks

Compositional Theory of Anaphora

- DTS provides a unified analysis for (general) inferences and anaphora resolution mechanisms (at least) for:
  - Deictic use and coreference
  - Bound variable anaphora (BVA)
  - E-type anaphora
  - Donkey anaphora
  - Bridging anaphora
  - Syllogistic anaphora
  - Disjunctive antecedents

Compositional Theory of Anaphora

- The background theory for DTS is an extention of DTT with underspecified terms and the @-rule.
  - Lexical items of anaphoric expressions and presupposition triggers are represented by using underspecified terms.
  - Context retrieval in DTS reduces to type checking.
  - Anaphora resolution and presupposition binding in DTS reduces to proof search.
  - @-elimination translates a proof diagram of DTS into a proof diagram of DTT, by which an SR in DTT is obtained with all anaphora resolved.

Natural language semantics via dependent types: Classics

- Donkey anaphora: Sundholm (1986)
- Translation from DRS to dependent type representations: Ahn and Kolb (1990)
- Summation: Fox (1994a,b)
- Ranta's TTG (Relative and Implicational Donkey Sentences, Branching Quantifiers, Intensionality, Tense): Ranta (1994)
- Translation from Montague Grammar to dependent type representations: Dávila-Pérez (1995)
- Presupposition Binding and Accommodation, Bridging: Krahmer and Piwek (1999), Piwek and Krahmer (2000)

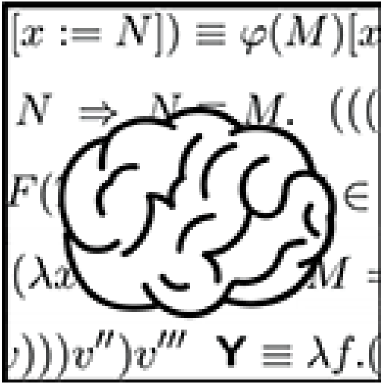
Natural language semantics via dependent types: Recent frameworks

- Type Theory with Record (TTR): Cooper (2005)
- Modern Type Theory: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- Semantics with Dependent Types: Grudzinska and Zawadowski (2014; 2017)
- **Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)**
- (Dynamic Categorial Grammar: Martin and Pollard (2014))

Semantic Analyses by DTS

- Generalized Quantifiers: Tanaka (2014)
- Honorification: Watanabe et al. (2014)
- Conventional Implicature: Bekki and McCready (2015)
- Factive Presuppositions: Tanaka et al. (2015)(2017)
- Dependent Plural Anaphora: Tanaka et al. (2017)
- Paycheck sentences: Tanaka et al. (2018) in NLCS2018 (July 7)
- Comparision with DRT: Yana et al. (forthcoming)
- Social meaning of accommodation: Ito et al. (forthcoming)
- Coercion and Metaphor: Kinoshita et al. (forthcoming)

Thank you!



Typing Rules in DTS

Axioms and Structural Rules

$$\frac{A : \mathsf{type}_i}{x : A}\,(VAR) \qquad \frac{(c : A) \in \sigma}{c : A}\,(CON) \qquad \frac{}{\mathsf{type}_i : \mathsf{type}_{i+1}}\,(\mathsf{type}F)$$

$$\frac{M : A \quad N : B}{M : A}\,(WK) \qquad \frac{M : A \quad A =_\beta B}{M : B}\,(CONV)$$

Π-type F/I/E rules

$$\dfrac{A : \mathsf{type}_i \qquad \begin{array}{c}\overline{x : A}^{\,k} \\ \vdots \\ B : \mathsf{type}_j\end{array}}{(x : A) \to B : \mathsf{type}_{max(i,j)}}\ (\Pi F),k$$

$$\dfrac{A : \mathsf{type}_i \qquad \begin{array}{c}\overline{x : A}^{\,k} \\ \vdots \\ M : B\end{array}}{\lambda x.M : (x : A) \to B}\ (\Pi I),k \qquad\qquad \dfrac{M : (x : A) \to B \qquad N : A}{MN : B[N/x]}\ (\Pi E)$$

Σ-type F/I/E rules

$$\dfrac{A : \mathsf{type}_i \quad \begin{array}{c}\overline{x : A}^{\,k} \\ \vdots \\ B : \mathsf{type}_j\end{array}}{(x : A) \times B : \mathsf{type}_{max(i,j)}}\ (\Sigma F), k$$

$$\dfrac{M : A \quad N : B[M/x]}{(M, N) : (x : A) \times B}\ (\Sigma I)$$

$$\dfrac{M : (x : A) \times B}{\pi_1(M) : A}\ (\Sigma E) \qquad \dfrac{M : (x : A) \times B}{\pi_2(M) : B[\pi_1(M)/x]}\ (\Sigma E)$$

Disjoint Union Type F/I/E rules

$$\frac{A : \text{type} \quad B : \text{type}}{A \uplus B : \text{type}}\ (\uplus F)$$

$$\frac{M : A}{\iota_1(M) : A \uplus B}\ (\uplus I) \qquad \frac{N : B}{\iota_2(N) : A \uplus B}\ (\uplus I)$$

$$\frac{L : A \uplus B \quad C : (A \uplus B) \longrightarrow \text{type} \quad \begin{array}{c} \overline{x : A}^{\,k} \\ \vdots \\ M : C(\iota_1(x)) \end{array} \quad \begin{array}{c} \overline{x : B}^{\,k} \\ \vdots \\ N : C(\iota_2(x)) \end{array}}{\text{case of } (L; \lambda x.M)\lambda x.N : C(L)}\ (\uplus$$

Natural Number Type F/I/E rules

$$\frac{}{\mathbb{N} : \mathsf{type}}\ (\mathbb{N}F)$$

$$\frac{}{0 : \mathbb{N}}\ (\mathbb{N}I) \qquad \frac{n : \mathbb{N}}{\mathsf{s}(n) : \mathbb{N}}\ (\mathbb{N}I)$$

$$\frac{n : \mathbb{N} \quad C : \mathbb{N} \to \mathsf{type} \quad e : C(0) \quad \begin{array}{c} \overline{x : \mathbb{N}}^{k} \quad \overline{y : C(x)}^{k} \\ \vdots \\ M : C(\mathsf{s}(x)) \end{array}}{\mathsf{natrec}(n, e, \lambda x.\lambda y.M) : C(n)}\ (\mathbb{N}E), k$$

Enumeration Type F/I/E rules

$$\frac{}{\{a_1, \ldots, a_n\} : \mathsf{type}}\ (\{\}F)$$

$$\frac{}{a_i : \{a_1, \ldots, a_n\}}\ (\{\}I)$$

$$\frac{M : \{a_1, \ldots, a_n\} \quad C : \{a_1, \ldots, a_n\} \to \mathsf{type} \quad N_1 : C(a_1) \quad \ldots \quad N_n : C(a_n)}{\mathsf{case}_M(N_1, \ldots, N_n) : C(M)}\ (\{\}E)$$

Intensional Equality Type F/I/E rules

$$\frac{A : \mathsf{s} \quad M : A \quad N : A}{M =_A N : \text{type}}\ (\text{Id}F)$$

$$\frac{A : \text{type} \quad M : A}{\text{refl}_A(M) : M =_A M}\ (\text{Id}I)$$

$$\frac{E : M_1 =_A M_2 \quad C : (x : A) \to (y : A) \to (x =_A y) \to \text{type} \quad N : (x : A) \to Cxx(\text{refl}_A(x)}{\text{idpeel}(e, N) : CM_1M_2E}$$

@-rule

$$\frac{A : \text{type} \quad A \ \textit{true}}{@_i A : A}\ (@)$$

@-Elimination Rules in DTS

Axioms

$$\left[\!\!\left[\dfrac{A : \mathsf{s} \quad \overset{\mathcal{D}_M}{M : A'}}{@_i A : A} (@) \right]\!\!\right] = \overset{\mathcal{D}_M}{M : A'}$$

$$\left[\!\!\left[\dfrac{}{c : A} (CON) \right]\!\!\right] = \dfrac{}{c : A} (CON)$$

$$\left[\!\!\left[x : A \right]\!\!\right] = x : A$$

$$\left[\!\!\left[\dfrac{}{\mathsf{type} : \mathsf{kind}} (\mathsf{type}F) \right]\!\!\right] = \dfrac{}{\mathsf{type} : \mathsf{kind}} (\mathsf{type}F)$$

Π-type

$$\left[\!\!\left[\dfrac{\begin{array}{cc} & x : A' \\ \mathcal{D}_A & \mathcal{D}_B \\ A : \mathsf{s}_1 & B : \mathsf{s}_2 \end{array}}{(x : A) \to B : \mathsf{s}_2} (\underline{\Pi F}) \right]\!\!\right] = \dfrac{\begin{array}{cc} & x : A' \\ [\![\mathcal{D}_A]\!] & [\![\mathcal{D}_B]\!] \\ A' : \mathsf{s}_1 & B' : \mathsf{s}_2 \end{array}}{(x : A') \to B' : \mathsf{s}_2} (\Pi F)$$

$$\left[\!\!\left[\dfrac{\begin{array}{cc} & x : A' \\ \mathcal{D}_A & \mathcal{D}_M \\ A : \mathsf{s}_1 & M : B \end{array}}{\lambda x.M : (x : A) \to B} (\underline{\Pi I}) \right]\!\!\right] = \dfrac{\begin{array}{cc} & x : A' \\ [\![\mathcal{D}_A]\!] & [\![\mathcal{D}_M]\!] \\ A' : \mathsf{s}_1 & M' : B' \end{array}}{\lambda x.M' : (x : A') \to B'} (\Pi I)$$

$$\left[\!\!\left[\dfrac{\begin{array}{cc} \mathcal{D}_M & \mathcal{D}_N \\ M : (x : A) \to B & N : A \end{array}}{MN : B'} (\underline{\Pi E}) \right]\!\!\right] = \dfrac{\begin{array}{cc} [\![\mathcal{D}_M]\!] & [\![\mathcal{D}_N]\!] \\ M' : (x : A') \to B' & N' : A' \end{array}}{M'N' : B''} (\Pi E)$$

where $B'[N'/x] \twoheadrightarrow_\beta B''$

Σ-type

$$\left[\!\!\left[\dfrac{\overset{\mathcal{D}_A}{A : \mathsf{s}_1} \quad \overset{x : A'}{\overset{\mathcal{D}_B}{B : \mathsf{s}_2}}}{\begin{bmatrix} x{:}A \\ B \end{bmatrix} : \mathsf{s}_2}(\Sigma F)\right]\!\!\right] = \dfrac{\overset{[\![\mathcal{D}_A]\!]}{A' : \mathsf{s}_1} \quad \overset{x : A'}{\overset{[\![\mathcal{D}_B]\!]}{B' : \mathsf{s}_2}}}{\begin{bmatrix} x{:}A' \\ B' \end{bmatrix} : \mathsf{s}_2}(\Sigma F)$$

$$\left[\!\!\left[\dfrac{\overset{\mathcal{D}_M}{M : A} \quad \overset{\mathcal{D}_N}{N : B'}}{(M, N) : \begin{bmatrix} x{:}A \\ B \end{bmatrix}}(\Sigma I)\right]\!\!\right] = \dfrac{\overset{[\![\mathcal{D}_M]\!]}{M' : A'} \quad \overset{[\![\mathcal{D}_N]\!]}{N' : B'''}}{(M', N') : \begin{bmatrix} x{:}A' \\ B'' \end{bmatrix}}(\Sigma I)$$

$$\text{where } B''[M'/x] \twoheadrightarrow_\beta B'''$$

$$\left[\!\!\left[\dfrac{\overset{\mathcal{D}_M}{M : \begin{bmatrix} x{:}A \\ B \end{bmatrix}}}{\pi_1(M) : A}(\Sigma E)\right]\!\!\right] = \dfrac{\overset{[\![\mathcal{D}_M]\!]}{M' : \begin{bmatrix} x{:}A' \\ B' \end{bmatrix}}}{\pi_1(M') : A'}(\Sigma E)$$

$$\left[\!\!\left[\dfrac{\overset{\mathcal{D}_M}{M : \begin{bmatrix} x{:}A \\ B \end{bmatrix}}}{\pi_2(M) : B'}(\Sigma E)\right]\!\!\right] = \dfrac{\overset{[\![\mathcal{D}_M]\!]}{M' : \begin{bmatrix} x{:}A' \\ B'' \end{bmatrix}}}{\pi_2(M') : B'''}(\Sigma E)$$

$$\text{where } B''[\pi_1(M')/x] \twoheadrightarrow_\beta B'''$$

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