3 Myths about Predicate Modal Logic

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Outline

Problem

The following took on an essentially stable form prior to 1950:

- Predicate logic
- Propositional modal logic (say, $S4$)

So, what about ($S4$) **predicate modal logic**?

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So, what about ($S4$) **predicate modal logic**?

- Studied since Barcan (1946).
- Raises some tricky issues.
- To see this, it helps to review Quine's criticisms of the endeavor.

Outline

Quine on Modal Logic

"Reference and Modality" (1953-1980)

Quine notes the following apparent failure of substitution of equals for equals:

$$\frac{\text{9 is necessarily greater than 7.} \qquad \text{The number of planets is 9.}}{\text{The number of planets is necessarily greater than 7.}} \text{ ✗}$$

So, says Quine, "x is necessarily greater than 7" cannot be a property of objects. If anything, a property of descriptions of objects.

Quine on Modal Logic (continued)

"Reference and Modality" (1953-1980)

Apparently, "x is necessarily greater than 7" is a property of descriptions of objects.

But then is "Something is necessarily greater than 7", *i.e.* $\exists x.\Box(x > 7)$, to mean "Some description of an object is necessarily greater than 7"?

Quine on Modal Logic (continued)

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This entails a non-uniform and exotic analysis of quantifiers, which Quine rejects.

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...the important point to observe is that granted an understanding of the modalities..., and given an understanding of quantification ordinarily so called, we do not come out automatically with any meaning for quantified modal logic sentences....

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In a word, Quine wants *modularity*.

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Historically, approaches indeed do one (Barcan 1990, *etc.*) or the other (Montague 1973, Fitting and Mendelsohn 1998, Garson 2013, *etc.*).

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He predicts that modal logic will, rather, force changes to the predicate logic treatment of terms or quantifiers.

Historically, approaches indeed do one (Barcan 1990, *etc.*) or the other (Montague 1973, Fitting and Mendelsohn 1998, Garson 2013, *etc.*).

These last exemplify a standard approach, which has led to some “myths”.

Outline

Myth 1

We use, in this section, the syntax of the pioneering Montague (1973).

Myth 1

In the context of a modal operator, substitution of equals for equals must fail.

The invalidity of the inference

$$\frac{\Box(9 > 7) \qquad n = 9}{\Box(n > 7)} \text{ ✗}$$

represents a failure of substitution of equals for equals within the system.

Myth 2

Myth 2

In the context of a modal operator, ordinary quantifier rules such as existential generalization must fail.

The invalidity of the inference

$$\frac{\Box G(p)}{\exists x.\Box G(x)} \text{ ✗}$$

represents a failure of existential generalization within the system.

Myth 3

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De re is the result of a modal operator occurring inside the scope of a quantifier or other scope-taking operator.

This is the **scope theory** of *de dicto*/*de re*.

	De Dicto	*De Re*
existential quantifier	$\Box\exists x.G(x)$	$\exists x.\Box G(x)$
constant	$\Box(n > 7)$	$(\lambda x.\Box(x > 7))n$

Table: *De Dicto*/*De Re* in Montague

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But note that for Montague, *e.g.* $\Box(x > 7)$ is *de re*, *contra* Myth 3.

And, therefore, the meaning of the *de dicto* $\Box(n > 7)$ is not a function of $\Box(x > 7)$'s. Yikes!

Outline

An Approach from Type Theory

Some work in modal type theory (Bierman and de Paiva 2000, Pfenning and Davies 2001, *etc.*) suggests how to make a system of predicate modal logic that avoids Myths 1-3.

Zwanziger (2017) gives a syntax which does this. Though not the most refined, it is a particularly simple implementation of the lessons of the modal type theory approach.

Zwanziger (2017)

General Idea

- *De re* is marked by square brackets: $\Box([n] > 7)$ *vs.* $\Box(n > 7)$.
- In the context of a $\Box$, the variables are *de re*: $\Box([x] > 7)$, never $\Box(x > 7)$!
- Everything from predicate logic ($\forall$, $\exists$, λ, $=$, ...) is as usual.

Let's see how this works.

Myth 1, Revisited

The felicitous *de re* inference

$$\frac{\Box([9] > 7) \qquad n = 9}{\Box([n] > 7)} \checkmark$$

goes thru, but the aberrant *de dicto* one

$$\frac{\Box(9 > 7) \qquad n = 9}{\Box(n > 7)} \text{✗}$$

does not, as $\Box(x > 7)$ is not a formula!

Myth 2, Revisited

The felicitous *de re* inference

$$\frac{\Box G[p]}{\exists x.\Box G[x]} \checkmark$$

goes thru, but the aberrant *de dicto* one

$$\frac{\Box G(p)}{\exists x.\Box G(x)} \times$$

does not, as $\Box G(x)$ is not a formula (nor $\exists x.\Box G(x)$)!

Myth 3, Revisited

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Myth 3, Revisited

Recall that, already in Montague, the scope theory could not explain *de dicto*/*de re* for open formulas.

Here, similarly, we cannot attribute *de re* in $\Box([x] > 7)$ to any operator binding x.

Rather, *de re* is (always) the result of applying a modal operator to an open formula.

Principle A

The general lesson for modal logic from modal type theory is that Myths 1-3 can (and thus should) be avoided. Furthermore, Quine's demands for modularity can (and thus should) be met. The general strategy for doing this, used in the foregoing discussion, is:

Principle A

In the context of a modal operator, all free variables will receive *de re* interpretation, and should be marked as such.

Outline

Types

Following are some details of (a simplified version of) Zwanziger (2017):

- There are basic types E, T.
- If A and B are types, then so is $A \to B$.

The type $\mathsf{E} \to \mathsf{T}$ is for predicates on entities, $(\mathsf{E} \to \mathsf{T}) \to \mathsf{T}$ for predicates on predicates on entities, etc.

Term Calculus

(Non-Modal) Logic

- $\dfrac{}{\cdot \mid \top : \mathrm{T}}$, and similarly for $\bot$
- $\dfrac{\phi : \mathrm{T} \qquad \psi : \mathrm{T}}{\phi \wedge \psi : \mathrm{T}}$, and similarly for $\vee$ and $\Rightarrow$.
- $\dfrac{\phi : \mathrm{T}}{\neg\phi : \mathrm{T}}$
- $\dfrac{\Gamma, x : A \mid \phi : \mathrm{T}}{\Gamma \mid \forall x.\phi : \mathrm{T}}$, and similarly for $\exists$.
- $\dfrac{t : A \qquad u : A}{t =_A u : \mathrm{T}}$

Term Calculus

Variables and Function Types

- $$\frac{}{\Gamma, x : A, \Delta \mid x : A}$$
- $$\frac{\Gamma, x : A \mid t : B}{\Gamma \mid \lambda x.t : A \to B}$$
- $$\frac{t : A \to B \qquad u : A}{tu : B}$$

Term Calculus

Modality

($\Box$ Form.).

$$\frac{\Gamma \mid s_1 : A_1, \ldots, \Gamma \mid s_n : A_n \qquad x_1 : A_1, \ldots, x_n : A_n \mid \phi(x_1, \ldots, x_n) : \mathsf{T}}{\Gamma \mid \Box\phi([s_1], \ldots, [s_n]) : \mathsf{T}}$$

Entailment

Logic

- We axiomatize entailment as a judgement of form

$$\Gamma \mid \phi \vdash \psi$$

- Rules for (non-modal) logic are as usual
- In particular, existential generalization and universal instantiation are valid.

Entailment

Modality

(ML1). $$\frac{\phi(x) \vdash \psi(x)}{\Box\phi([x]) \vdash \Box\psi([x])}$$

(ML2). $\Box\phi([x]) \vdash \phi(x)$

(ML3). $\Box\phi([x]) \vdash \Box\Box\phi([[x]])$

(ML4). $\Box\phi([x]) \wedge \Box\psi([x]) \vdash \Box(\phi \wedge \psi)([x])$

(ML5). $\top \vdash \Box\top$

Entailment

Modality (continued)

In particular the following principles are derivable:

(Necessitation). $$\frac{\top \vdash \phi(x)}{\top \vdash \Box\phi([x])}$$

(K). $$\Box(\phi \Rightarrow \psi)([x]) \vdash \Box\phi([x]) \Rightarrow \Box\psi([x])$$

(Converse Barcan Principle). $$\frac{\phi(x, y) : \mathrm{T}}{\Box(\forall y.\phi)([x]) \vdash \forall y.\Box\phi([x, y])}$$

Outline

Conclusion and Future Prospects

- It is possible for predicate modal logic to avoid Myths 1-3 and, furthermore, meet Quine's demands for modularity.
- There is a need for further refinements which maintain these advantages.
- Likely relevant: recent work on modal dependent type theory (Nanevski *et al.* 2008, Shulman 2018) and adjoint dependent type theory (Krishnaswami *et al.* 2015, Zwanziger 2019).

Thanks for your attention!